VIDYA BHAWAN BALIKA VIDYA PITH

शक्तिउत्थानआश्रमलखीसरायबिहार

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Property I

(i) $\sin^{-1}(\sin\theta) = \theta$,   if $\theta \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta$;   if $\theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta$;   if $\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(iv) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$; if $\theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right], \theta \neq 0$

(v) $\sec^{-1}(\sec\theta) = \theta$;   if $\theta \in [0, \pi], \theta \neq \dfrac{\pi}{2}$

(vi) $\cot^{-1}(\cot\theta) = \theta$;   if $\theta \in (0, \pi)$

Property II

(i) $\sin(\sin^{-1}x) = x$;   if $x \in [-1, 1]$

(ii) $\cos(\cos^{-1}x) = x$;   if $x \in [-1, 1]$

(iii) $\tan(\tan^{-1}x) = x$;   if $x \in R$

(iv) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(v) $\sec(\sec^{-1}x) = x$;   if $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot(\cot^{-1}x) = x$;   if $x \in R$

Property III

(i) $\sin^{-1}(-x) = -\sin^{-1}x$;   if $x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$;   if $x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$;   if $x \in R$

(iv) $\operatorname{cosec}^{-1}(-x) = \pi - \operatorname{cosec}^{-1}x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$;   if $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x$;   if $x \in R$

Property IV

(i) $\sin^{-1}\left(\dfrac{1}{x}\right) = \operatorname{cosec}^{-1}x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(ii) $\cos^{-1}\left(\dfrac{1}{x}\right) = \sec^{-1}x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(iii) $\tan^{-1}\left(\dfrac{1}{x}\right) = \begin{cases} \cot^{-1}x; & \text{if } x > 0 \\ -\pi + \cot^{-1}x; & \text{if } x < 0 \end{cases}$

Property V

(i) $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$; if $x \in [-1, 1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}$; if $x \in R$

(iii) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \dfrac{\pi}{2}$; if $x \in (-\infty, -1] \cup [1, \infty)$

Property VI

(i) $\sin^{-1} x + \sin^{-1} y = \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}; \\ \pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}; \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}; \end{cases}$

if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$ or if $xy < 0$ and $x^2 + y^2 > 1$

if $0 < x, y \le 1$ and $x^2 + y^2 > 1$

if $-1 \le x, y < 0$ and $x^2 + y^2 > 1$

(ii) $\sin^{-1} x - \sin^{-1} y = \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}; \\ \pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}; \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}; \end{cases}$

if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$ or if $xy > 0$ and $x^2 + y^2 > 1$

if $0 < x \le 1, -1 \le y \le 0$ and $x^2 + y^2 > 1$

if $-1 \le x < 0, 0 < y \le 1$ and $x^2 + y^2 > 1$

Property VII

(i) $\cos^{-1} x + \cos^{-1} y$

$= \begin{cases} \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}; & \text{if } -1 \le x, y \le 1 \text{ and } x + y \ge 0 \\ 2\pi - \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}; & \text{if } -1 \le x, y \le 1 \text{ and } x + y \le 0 \end{cases}$

(ii) $\cos^{-1} x - \cos^{-1} y$

$= \begin{cases} \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}; & \text{if } -1 \le x, y \le 1 \text{ and } x \le y \\ -\cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}; & \text{if } -1 \le y \le 0, 0 < x \le 1 \text{ and } x \ge y \end{cases}$

Property VIII

(i) $\tan^{-1} x + \tan^{-1} y$

$= \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right); & \text{if } xy < 1 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right); & \text{if } x > 0, y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right); & \text{if } x < 0, y < 0 \text{ and } xy > 1 \end{cases}$

(ii) $\tan^{-1} x - \tan^{-1} y$

$= \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right); & \text{if } xy > -1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right); & \text{if } x > 0, y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right); & \text{if } x < 0, y > 0 \text{ and } xy < -1 \end{cases}$

Property IX

(i) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}} = \cot^{-1}\dfrac{\sqrt{1-x^2}}{x}$

$= \sec^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\dfrac{1}{x}\right)$

(ii) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$

$= \cot^{-1}\dfrac{x}{\sqrt{1-x^2}} = \sec^{-1}\left(\dfrac{1}{x}\right)$

$= \operatorname{cosec}^{-1}\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

(iii) $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\dfrac{1}{x}\right)$

$= \operatorname{cosec}^{-1}\left(\dfrac{\sqrt{1+x^2}}{x}\right) = \sec^{-1}(\sqrt{1+x^2})$

Property X

(i) $2\sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}} \\[2mm] \pi - \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } \dfrac{1}{\sqrt{2}} \le x \le 1 \\[2mm] -\pi - \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } -1 \le x \le -\dfrac{1}{\sqrt{2}} \end{cases}$

(ii) $2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2-1); & \text{if } 0 \le x \le 1 \\[2mm] 2\pi - \cos^{-1}(2x^2-1); & \text{if } -1 \le x \le 0 \end{cases}$

(iii) $2\tan^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); & \text{if } -1 < x \le 1 \\[2mm] \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); & \text{if } x > 1 \\[2mm] -\pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); & \text{if } x < -1 \end{cases}$

Property XI

(i) $3 \sin^{-1} x = \begin{cases} \sin^{-1}(3x - 4x^3); & \text{if } -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ \pi - \sin^{-1}(3x - 4x^3); & \text{if } \dfrac{1}{2} < x \le 1 \\ -\pi - \sin^{-1}(3x - 4x^3); & \text{if } -1 \le x < -\dfrac{1}{2} \end{cases}$

(ii) $3 \cos^{-1} x = \begin{cases} \cos^{-1}(4x^3 - 3x); & \text{if } \dfrac{1}{2} \le x \le 1 \\ 2\pi - \cos^{-1}(4x^3 - 3x); & \text{if } -\dfrac{1}{2} \le x \le \dfrac{1}{2} \\ 2\pi + \cos^{-1}(4x^3 - 3x); & \text{if } -1 \le x \le -\dfrac{1}{2} \end{cases}$

(iii) $3 \tan^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right); & \text{if } -\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right); & \text{if } x > \dfrac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right); & \text{if } x < -\dfrac{1}{\sqrt{3}} \end{cases}$